

A Logic of Topic-sensitive and Fragmented Hyperintensional Belief

Abstract

Traditional Hintikka-style epistemic logics [13, 12] can lead to undesired results when modeling the reasoning of realistic human agents. One such undesired result is called the problem of logical omniscience: agents modelled via Hintikka-style epistemic logics know or believe all tautologies and all logical consequences of their knowledge or belief. Human agents cannot possess such unrealistic knowledge or beliefs, since they only have limited computational, conceptual and reasoning capacities, as well as finite time restrictions [19, 7].

To address the logical omniscience problem, various logics have been proposed, including awareness logics [8, 9], impossible worlds semantics [18, 15] and topic-sensitive frameworks [2, 4, 17, 10, 5]. These logics are hyperintensional, in the sense that they allow for distinctions between the propositions that may be true in the same possible worlds but are not equivalent due to some reasons.

However, no universally accepted theory has resolved all forms of logical omniscience. For instance, fragmentationalism [14, 6, 21, 22] challenges many hyperintensional theories like awareness logics by arguing that assuming an agent possesses a single coherent belief or knowledge system at all times is overly idealized. Human agents' epistemic states can be fragmented into different parts and they may use only one part or several parts in one situation. Additionally, previous hyperintensional theories such as impossible worlds semantics and topic-sensitive frameworks cannot deal well with issues concerning epistemic modalities, especially the ones concerning alethic modal operators within doxastic contexts. This can give rise to various problems, including the invalidity of epistemic contradiction ($B\phi \wedge B\Diamond\neg\phi \vDash \perp$) [20, 11, 1] and the undesired inference from a belief of a necessity to the necessity itself.

This research aims to introduce a novel doxastic logic, which addresses the challenges faced by the aforementioned accounts of hyperintensionality. We use the notion of topic-relevance to achieve hyperintensionality, modeling belief based on the fragments generated by topics and information pieces. We adopt a qualitative account of belief, based on a relational Kripke-style semantics as in Hintikka-style epistemic logics, while integrating the idea of fragmentationalism and topic-sensitivity of intentional mental states [3].

The resulting logical framework improves on the aforementioned frameworks by solving more aspects of the problems of logical omniscience and epistemic modality and, moreover, by addressing some of the criticisms fragmentationalism has received. Our main contribution consists in the following two features of the proposed formalism:

The first one is that we provide a formal framework to achieve the objectives suggested by fragmentationalism. We propose that an agent's belief states are divided into fragments based on subject matter, and it is from these fragments that beliefs arise. To believe that a proposition is true, the agents only need to concentrate on the information pieces they possess that are relevant to the proposition – *the relevant fragments of our belief state*. This implies that only specific portions of our belief state are utilized in evaluating a proposition, indicating that the epistemic belief is fragmented. Additionally, we use topics to determine this kind of relevance, which means that topics can segment the belief state into fragments, thereby gaining the anticipated benefits of fragmentation without introducing it as a primitive element in the model. In this way, the fragmentation in our logic is natural and not pre-setted, bypassing the criticism that fragmentation is ad hoc, as in [16].

This approach also allows for a unique, yet intuitive interpretation of necessity operators within doxastic contexts, interpreting “Believe that A is necessary” as “Believe A after considering it very

carefully”. While to believe that a proposition is true we only need to check the relevant parts of our belief state, to believe that a proposition is necessary we need to check the whole belief state, exhausting our epistemic ability. The latter requires that the proposition is topic relevant to all the parts of the belief state, which can be also modeled naturally in our framework.

Therefore, our logic not only addresses the issues of logical omniscience but also tackles those arising from fragmentation and epistemic modality.

The proposed formal semantics for the corresponding logic will be presented in detail during the talk.

References

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