Inductive learning with first-order justification logic

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1 Introduction

The project of 'neural-symbolic integration' could be understood as an attempt to find a precise qualitative representation of *uncertainty* resulting from inductive learning and generalization. At the level of qualitative representation, one way to represent uncertainty is through a system of general rules with exceptions [3, p. 12401]. In this paper, I develop one such system based on a non-monotonic variant of first-order justification logic proposed in [1].

Building on the work in [4, 5], where default rules are given for propositional justification logic, the first-order extension enables us to formalize default schemes. In [4, 5], the only type of rules are exclusively specific defaults, which may be seen as a limitation on their generalization capabilities. Introducing default schemes brings us closer to the original motivation behind the standard default logics [6], which was already considered as a 'reinvention' of inductive, statistical reasoning [7]. The added values of justification logic is that it is expressive enough to formalize rules with exceptions in the object-language and that it can provide explanations for its inference process.

Standard justification logic formulas $t : F$ are interpreted as justification assertions of the type 't is a justification of F'. The distinctive feature of the language of first-order justification logic is that it allows formulas of the type $t :_X F$, where t is a proof term, X is a finite set of individual variables, and F is a formula. We will assume axioms and inference rules of the first-order variant of the logic of proofs [2, p. 227].[1] These axioms and rules are inherently monotonic and we use them to describe certain information and fully-specified states of affairs. Based on the grammar of first-order justification logic, we introduce the following default rule logic schemes with justifications:

$$\frac{t_{\{x\}} : F :: (u \cdot t)_{\{x\}} : G}{(u \cdot t)_{\{x\}} : G},$$

where x in $t_{\{x\}} : F$ is a free variable throughout the derivation t.

Such rules enable us to encode the statistical and quantitative characteristics of the data as weights of the default rules. We define default theories based on default rule schemes and their pertaining weights. Then we define a two-layered semantics for such theories. The first part of the semantics is the process of learning and data-dependent generalizing to determine the weights of justification terms. The second part is reasoning with those weights by applying the calculus of weighted reasons utilizing a generalization made in a default scheme to a specific element from the domain of objects. I argue that this two-layered process describes an intuitive

[1] Excluding the axiom schema B4 that regulates the behavior of the 'proof checker' operator '!'.

and plausible 'neural-symbolic' architecture that combines learning and reasoning in first-order default justification logic.

References

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