Arbitrary Abstraction and Logicality

Abstractionist theories are systems composed by a logical theory augmented with one or more abstraction principles (AP), of form: $f_R\alpha = f_R\beta \leftrightarrow R(\alpha, \beta)$ – that introduce, namely rule and implicitly define, the corresponding term-forming operators f_R. The issue of the logicality of these theories was originally raised into the seminal abstractionist program, Frege's Logicism – proposed with the foundational purpose to derive arithmetical laws as logical theorems and to define arithmetical expressions by logical terms. As is well-known, this project failed, but the issue of logicality represents, still today, an open question of the abstractionist debate (cf. [8], [6], [1], [4], [2], [5]). More precisely, given a semantical definition of logicality as isomorphism invariance, we are able to prove that some abstraction principles (like Hume's Principle) are logical ([4]) but their implicit *definienda* are not ([1])– so preventing a full achievement of Logicist goal.

My preliminary aim will consists in showing that this unfortunate situation closely depends on the (unjustified) adoption of a same notion of *canonical* reference for all the expressions of a same syntactical category (e.g. singular terms as always referential and denoting singular, knowable and standard objects). On the contrary, a less demanding reading of the abstractionist vocabulary is available: it is based on an arbitrary interpretation of the abstractionist vocabulary and turns out to be preferable as more faithful to the theory. My double aim will consist in inquiring the consequences of such an interpretation on the logicality of abstractionist theories both from a formal and from a philosophical point of view.

On the one side, from a formal point of view, given such an interpretation of the APs, we can rephrase the main criterion of logicality for abstraction operators (*objectual invariance*, cf. [1]), obtaining a weaker one (*weak objectual invariance*[1], WOI, cf. [9], [2]) and proving that it is satisfied not not only by cardinal operator but also by many other second-order ones, including those implicitly defined by consistent weakenings of Fregean Basic Law V. So, we will note that, given (what I argued as) a preferable reading of the APs, both main strategies pursued in the last century to save Fregean

[1]An expression ϕ is *weakly objectually invariant* just in case, for all domains D, D' and bijections ι from D to D', the set of candidate denotations of ϕ on D (ϕ^{*D}) = $\{\gamma : \gamma$ is a candidate denotation for ϕ on $D\}$ is such that $\iota(\phi^{*D}) = \phi^{*D'} = \{\gamma : \gamma$ is a candidate denotation for ϕ on $D'\}$.

project – Neologicism and consistent revisions of *Grundgesetze* – are able to achieve the desirable logicality objective. Further generalising, I will prove that the logicality criterion could be satisfied by a large range of APs but, at the same time, it is able to introduce interesting differences. More precisely, I will prove that WOI is not satisfied by any first-order abstraction principles (cf. [8], [9]) and, by comparing respective schemas of first-order and second-order APs, we will note that it mirrors a relevant distinction between same-order and different-order abstraction principles.

On the other side, from the philosophical point of view, I will focus on the role of arbitrariness as a condition for the adoption of the abovementioned logicality criterion. Particularly, while WOI seems to testify the unexpected availability of the Logicist goal, the arbitrary interpretation of the vocabulary actually includes semantical insights that are radically alternative to Logicism. In order to argue for this latter consideration, I will suggest to precise the two main meanings of the informal notion of arbitrariness (i.e. the epistemicist meaning and the semantical one) in a model-theoretic perspective, by means of, respectively, a choice-like semantics and a modal semantics. Given these semantical frameworks, we will note that the arbitrary interpretation not only gives a structuralist nuance to the reading of the abstraction (by emphasising the role rather than the nature of the abstract entities), but its models are clearly more compatible with a nominalist account of the theoretical terms rather then with a Platonist one. Particularly, an analogy between the arbitrary interpretation of the APs and the semantics of some eliminative Structuralist reconstructions of the scientific theories ([7]) will be illustrated.

References

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