RANDOM EXPANSIONS OF FINITE STRUCTURES WITH BOUNDED DEGREE

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We consider finite relational signatures $\tau \subseteq \sigma$, a sequence of finite base τ-structures $(\mathcal{B}_n : n \in \mathbb{N})$ the cardinalities of which tend to infinity and such that there is a fixed finite upper bound of the degree of (the Gaifman graph of) every $\mathcal{B}_n$. We let $\mathbf{W}_n$ be the set of all expansions to σ of $\mathcal{B}_n$ and we consider a probabilistic graphical model, a concept used in machine learning and artificial intelligence, to generate a probability distribution $\mathbb{P}_n$ on $\mathbf{W}_n$ for all n.

We use a many-valued "probability logic" with truth values in the unit interval to express probabilities within probabilistic graphical models and to express queries on $\mathbf{W}_n$. This logic uses aggregation functions (e.g. the average) instead of quantifiers and it can express all queries (on finite structures) that can be expressed with first-order logic since the aggregation functions maximum and minimum can be used to express existential and universal quantifications, respectively. The main results presented, which are found in [K], concern asymptotic elimination of aggregation functions (the analogue of almost sure elimination of quantifiers for two-valued logics with quantifiers) and the asymptotic distribution of truth values of formulas (the analogue of logical convergence results for two-valued logics).

From one point of view these results are a further development in the spirit of finding logical limit laws and results about "almost sure elimination of quantifiers" in finite model theory. But the inspiration and methodology of the present work comes more from previous studies of the author and Felix Weitkämper and of Manfred Jaeger [J] which in turn are inspired by problems related to artificial intelligence and machine learning.

[J] M. Jaeger, Convergence results for relational Bayesian networks, *Proceedings of the 13th Annual IEEE Symposium on Logic in Computer Science (LICS 98)* (1998).

[KW1] V. Koponen, F. Weitkämper, Asymptotic elimination of partially continuous aggregation functions in directed graphical models, *Information and Computation*, Vol. 293 (2023) 105061, `https://doi.org/10.1016/j.ic.2023.105061`.

[KW2] V. Koponen, F. Weitkämper, On the relative asymptotic expressivity of inference frameworks, `https://arxiv.org/abs/2204.09457`.

[K] Random expansions of finite structures with bounded degree, `https://arxiv.org/abs/2401.04802`.

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