Proof-Theoretic Considerations on the Structure of Reasoning with Counterfactuals and Knowledge

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Combined reasoning with counterfactuals and knowledge may involve constructions which: (i) embed knowledge (relativized to an agent a) in the antecedent (resp. succedent) of a counterfactual (e.g., 'If a were to know that A, a would believe that B'), or (ii) prefix knowledge to a counterfactual (e.g., 'a knows that if A were the case, B would be the case'), or (iii) do both; where the embedding and prefixing can be iterated.

The literature on the structural proof theory of the interaction of counterfactuals and knowledge does not abound. A rare example is [2], where a labelled (or external) sequent calculus for the logic of conditional belief (CDL; cf. [1]) is developed. CDL aims at modeling revisable belief. In particular, knowledge is here defined in terms of an operator for conditional belief which has essentially the meaning of the first example, but with 'know' replaced by 'learn'. As the subtitle of [2] suggests, the authors take a perspective on the logic and the semantics of counterfactuals and knowledge on which model-theoretic structure is methodologically fundamental. Specifically, the model-theoretic semantics is incorporated into the structural proof system.

In the talk, we take a perspective on which proof-theoretic structure is fundamental. Counterfactual inference will be construed as structural reasoning from counterfactual assumptions. Moreover, unlike [2] which is based on classical logic, we endorse a constructive conception of the meaning of counterfactuals and knowledge; specifically, a conception which appeals to canonical derivations. We combine components from [3] and [4], so as to obtain intuitionistic subatomic natural deduction systems for combined reasoning with ('would'- and 'might'-) counterfactuals and knowledge (resp. belief) which are proof-theoretically well-behaved (normalization, subexpression/subformula property, internal completeness) and which admit the formulation of a semantically autarkic proof-theoretic semantics for elementary combined constructions of the aforementioned kinds.

References

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