Maximally Substructural Classical Logic

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Roughly speaking, *structural principles* are principles whose formulation does not require any constant of the object language. Classical logic validates a number of well-known structural principles, such as for instance

$$\text{Id}\ \frac{}{A \dashv\!\vdash A} \qquad \text{Cut}\ \frac{\Gamma \dashv\!\vdash \Delta, A \qquad A, \Gamma \dashv\!\vdash \Delta}{\Gamma \dashv\!\vdash \Delta}$$

$$\text{C}{\dashv\!\vdash}\ \frac{A, A, \Gamma \dashv\!\vdash \Delta}{A, \Gamma \dashv\!\vdash \Delta} \qquad {\dashv\!\vdash}\text{C}\ \frac{\Gamma \dashv\!\vdash \Delta, A, A}{\Gamma \dashv\!\vdash \Delta, A}$$

$$\text{W}{\dashv\!\vdash}\ \frac{\Gamma \dashv\!\vdash \Delta}{A, \Gamma \dashv\!\vdash \Delta} \qquad {\dashv\!\vdash}\text{W}\ \frac{\Gamma \dashv\!\vdash \Delta}{\Gamma \dashv\!\vdash \Delta, A}$$

Here, $A, B, ...$ are formulas of the relevant object language, $\Gamma, \Delta, ...$ are multisets of formulas, and $\dashv\!\vdash$ is a dyadic relation representing logical consequence. 'C' stands for 'Contraction', 'W' for 'Weakening' and 'Id' for 'Identity'.

In the last years, we have seen the emergence of various logical systems that are coextensive with classical logic but in a certain sense invalidate some of the above structural principles. In the model-theoretic framework, these systems are obtained by finding a consequence relation that validates all and only the arguments that are valid in classical logic, but relative to which some of the structural principles are 'locally invalid', that is, fail to preserve satisfaction at every interpretation. Thus, for instance, Cobreros et. al. [1, 2] and Ripley [4, 5] present system **ST**, where Cut is locally invalid. Also, Rosenblatt [6] presents system **NC**, where not only Cut but also the principles of Contraction are locally invalid. Curiously, no system of this kind has been proposed were the principles of Weakening are locally invalid. Such a system seems possible in principle, since we know that there are sequent calculi for classical logic where the rules of Weakening are admissible but not derivable.

In this paper, we define a system whose consequence relation is coextensive with classical logic, but in which the principles of Cut, Contraction and Weakening are all locally invalid. We call this system **msCL** (for 'Maximally Substructural Classical Logic'), and obtain it by means of a six-valued semantics and a *sui generis* definition of logical consequence. We show that **msCL** can be used to provide a semantics for the sequent calculus **G3** for classical logic (see, e.g. Negri & von Plato [3]). Lastly, we show that the trick that we use to define **msCL** can be transposed to any other Tarskian logic. That is, for any Tarskian logic **L** it is possible to define a logic **msL** which is coextensive to **L** but locally invalidates all structural principles except for Id. This last result can be seen as a generalization of a recent work by Szmuc [7], who shows that for every Tarskian logic it is possible to define a coextensive system that locally invalidates Cut.

References

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