Imagination, Mereotopology, and Topic Expansion

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Joint work with **Aaron J. Cotnoir** (University of St Andrews).

Motivation - Imagination and its logic

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Imagination $\sim$ *Reality oriented mental simulation*.

This kind of imagination plays important roles in *counterfactual thought, pretence, contingency planing, decision making...*

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From daydreaming to decision-making, from pretending to planning, imagination plays a central role in many of the activities of everyday life... Though imagination is sometimes used to enable us to escape or look beyond the world as it is, as when we daydream or fantasize or pretend, it is also sometimes used to enable us to learn about the world as it is, as when we plan or make decisions or make predictions about the future. (Kind & Kung, 2019, p. 1, my emphasis)

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A number of empirically-motivated constraints on imagination (following Canavotto et al. 2020)[1]:

- Imagination is agentive and episodic.
- Acts of imagination have deliberate starting points, given by an input.
- Inputs are integrated with contextual background information.
- Imagination is constrained by topic and relevance.
- Imaginative acts are goal-driven and question-based.

See Berto (2018), *'Aboutness in Imagination'*.

[1] Also see Badura & Wansing (2020).

Berto's logic of imagination

It is a modal logic with a *binary modal operator interpreted as a variably strict modal with a topicality filter*.

- Acts of imagination have deliberate starting points, given by an input.

$I^{\varphi}\psi$: *In an act of imagination starting with input φ, one imagines that ψ.*

- Inputs are integrated with contextual background information.

(TC) $I^{\varphi}\psi$ is interpreted as a variably strict modal, in style of Stalnaker-Lewis conditionals via set-selection functions.

- Imagination is constrained by topic and relevance.

(AP) The topic of the input must be contained in the topic of the output.

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Intermezzo: Aboutness and Topicality

Aboutness is

'*the relation that meaningful items bear to whatever it is that they are* **on** *or* **of** *or that they* **address** *or* **concern**' (Yablo, 2014, p. 1),

namely their topic, or subject matter.

Declarative sentences are used to say true things about all kinds of topics. One says: "Kai is a logician". One thereby communicates something about *Kai's profession* and, more generally, *Kai*.

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thick proposition: $\langle$intension, topic or subject matter$\rangle$

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The **Boolean operators** are **topic-transparent**.

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X In the act of imagining people keep burning fossil fuels at this pace, Franz imagines that the polar ice will melt. (Berto & Özgün 2021, p. 3708)

Our Work

- AP is too strong![2]
- Imaginers should be free to move to other topics that are *connected* to the topics of the inputs, but not necessarily contained within them.
- We propose a new mereotopological approach to topics:

  the topic of imaginative outputs must be contained in an **expansion** *of the topic of the imaginative inputs.*

[2] Badura (2021) proposes a solution based on a first-order language and Hawke's issue-based theory of aboutness.

Our Work

- But this cannot be an **unconstraint** expansion!
  - *Preserve intuitive imaginative jumps to nearby topics*, as in the case of Gwenny or burning fossil fuel.
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- We explore the following topic-expansion operators:
  1. closure operator
  2. inclusive and monotone increasing operator
  3. inclusive and additive operator

Berto's logic of imagination - more formally

$$(\mathcal{L}) \quad \varphi := p \mid \neg\varphi \mid \varphi \wedge \varphi \mid \Box\varphi \mid I^{\varphi}\psi$$

A *ts-model* is a tuple $\mathcal{M} = \langle W, \{f_\varphi \mid \varphi \in \mathcal{L}\}, \mathcal{T}, \oplus, t, V \rangle$ where

1. W is a non-empty set of *possible worlds*;
2. $f_\varphi : W \times \mathcal{P}(W)$;
3. V : Prop $\to \mathcal{P}(W)$ is a valuation map;

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4. $\mathcal{T}$ is a non-empty set of *possible topics*;
5. $(\mathcal{T}, \oplus)$ is a *join semilattice*;
6. t : Prop $\rightarrow \mathcal{T}$ is the *topic function* which extends to the whole language as $t(\varphi) = t(p_1) \oplus \cdots \oplus t(p_n)$.

topic parthood: $a \leq b$ iff $a \oplus b = b$.

where $p_1, \ldots, p_n$ are the propositional variables occurring in φ.

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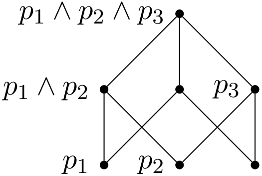

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$$(\mathcal{L}) \quad \varphi := p \mid \neg\varphi \mid \varphi \wedge \varphi \mid \Box\varphi \mid I^{\varphi}\psi$$

Given a ts-model $\mathcal{M} = \langle W, \{f_\varphi \mid \varphi \in \mathcal{L}\}, \mathcal{T}, \oplus, t, V\rangle$ and $w \in W$:

$\mathcal{M}, w \models I^{\varphi}\psi$ iff for all $w' \in W$(if $w' \in f_\varphi(w)$ then $\mathcal{M}, w' \models \psi$) and $t(\psi) \leq t(\varphi)$

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$$\mathcal{M}, w \models I^{\varphi}\psi \quad \text{iff} \quad f_\varphi(w) \subseteq [\![\psi]\!] \text{ and } t(\psi) \leq t(\varphi)$$

where $[\![\psi]\!] = \{w \in W : \mathcal{M}, w \models \psi\}$.

Our proposal - I

$$(\mathcal{L}) \quad \varphi := p \mid \neg\varphi \mid \varphi \wedge \varphi \mid \Box\varphi \mid I^{\varphi}\psi$$

Given a *topological* ts-model $\mathcal{M} = \langle W, \{f_{\varphi} \mid \varphi \in \mathcal{L}\}, \mathcal{T}, \oplus, c, t, V \rangle$ and $w \in W$:

$$\mathcal{M}, w \models I^{\varphi}\psi \quad \text{iff} \quad f_{\varphi}(w) \subseteq [\![\psi]\!] \text{ and } t(\psi) \leq c(t(\varphi))$$

where $c : \mathcal{T} \to \mathcal{T}$ is a topological closure operator on $(\mathcal{T}, \oplus)$.

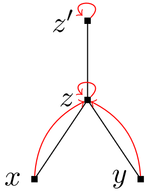
Our proposal - I

The closure operator

c satisfies for all $a, b \in \mathcal{T}$:

Inclusion	$x \leq c(x)$
Additivity	$c(x \oplus y) = c(x) \oplus c(y)$
Idempotence	$c(c(x)) = c(x)$

closure:



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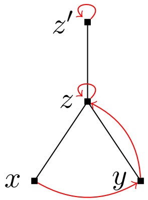

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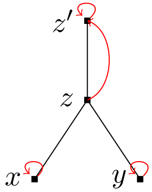
not additive:



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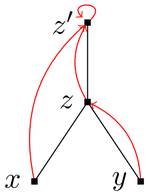

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Inclusion guarantees the closure of a topic is only ever an expansion.

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Additivity ensures that the result of closing the topic of a whole sentence φ is not different from the result of closing the topics of the atoms within φ and then fusing those topics.

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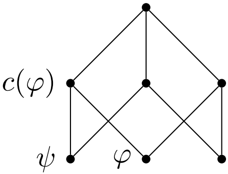
Additivity ensures that the result of closing the topic of a whole sentence φ is not different from the result of closing the topics of the atoms within φ and then fusing those topics.

Idempotence ensures that the expansion by imagination can't be repeated unless given different inputs.

Our proposal - I

$$\mathcal{M}, w \models I^{\varphi}\psi \quad \text{iff} \quad f_{\varphi}(w) \subseteq [\![\psi]\!] \text{ and } t(\psi) \leq c(t(\varphi))$$

Imaginative episodes can lead to proper expansions of the subject matter - *even to output topics that fail to mereologically overlap input topics*.



The "new" logic of closure semantics

The two semantics are equivalent:

Theorem

For all $\Gamma \subseteq \mathcal{L}$ *and* $\varphi \in \mathcal{L}$, $\Gamma \models \varphi$ *iff* $\Gamma \models_c \varphi$.

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When does the logic change?

Weaker expansion operators

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Nichols & Stich (2000) provide evidence that imaginative episodes of the sort under discussion display *non-inferential embellishment* of an imagined scenario. That is, the output of an imaginative episode is typically an expansion to contents:

> *that are not dictated by the pretense premise, or by the scripts and background knowledge that the pretender brings to the pretense episode. (Nichols & Stich, 2000, p.127)*

Weaker expansion operators

Inclusion $x \leq g(x)$ ✓

Additivity $g(x \oplus y) = g(x) \oplus g(y)$???

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Against additivity

What is imagined in an act of imagining the input φ *can go beyond* the totality of what is imagined in an act of imagining the atoms within φ separately.

Against additivity - example

Consider now that Helena lives in New York City and she has a friend named John who often moves from one city to another because of his job.

In an act of imagining that she is on her way to meet John, she imagines the activities she will be doing with John (having lunch, going to the movies etc.)

In an act of imagining that John is currently residing in Boston, she imagines how much John likes Boston, how cold Boston is in winter etc.

However, in an act of imagining that she is on her way to meet John and John is currently residing in Boston, she imagines that she is driving to New England.

Against monotone increasingness

Where topic x is part of topic y, there is some question as to whether the permitted embellishments of x are thereby contained within the permitted embellishments of y.

Some expansions of subject matter rule out, or at least makes unlikely, embellishments permitted by parts.

Against monotone increasingness - example

In an act of imagining Laura is 35 year-old woman running for a seat in the State Senate, you imagine that she wins on a platform of supporting gun control.

What if the input was "Laura is a 35 year-old woman running for a seat in the State Senate and her campaign was financed by the National Rifle Association"?

Against monotone increasingness - example

In an act of imagining John's dog bit him, you imagine that John's dog shows unwarranted aggressive behaviour.

What if the input was "John's dog bit him while John was trying to bite his dog"?

Our proposal - II

Given a ts-model *with functions* $\mathcal{M} = \langle W, \{f_\varphi \mid \varphi \in \mathcal{L}\}, \mathcal{T}, \oplus, g, t, V\rangle$ and $w \in W$:

$$\mathcal{M}, w \models I^\varphi \psi \quad \text{iff} \quad f_\varphi(w) \subseteq [\![\psi]\!] \text{ and } t(\psi) \leq g(t(\varphi))$$

where $g : \mathcal{T} \to \mathcal{T}$ a function.

Notation: $\overline{\varphi} := \bigwedge_{p \in Var(\varphi)}(p \vee \neg p)$, where $Var(\varphi)$ is the set of propositional variables occurring in φ.

Some technical results - definability

- Inclusivity is defined by $I^{\varphi}\overline{\varphi}$.
- Monotone increasingness is defined by $I^{\psi}\overline{\varphi} \supset I^{\chi}\overline{\varphi}$ for $Var(\psi) \subseteq Var(\chi)$.
- Additivity is not definable in $\mathcal{L}$!

Some technical results - axiomatization

- A sound and strongly complete axiomatization of the logic of all ts-models with functions.
- A sound and strongly complete axiomatization of the logic of inclusive models.
- A sound and strongly complete axiomatization of the logic of inclusive and monotone increasing models.
- The logic of inclusive and additive models (models with the so-called preclosure operator) is strictly weaker than Berto's logic.
- *We do not have an axiomatization for the logic of preclosure models.*

A few more words on the logic of preclosure operator

Our logic invalidates the following formula, which is part of Berto's logic:

$$\psi := (I^{p}\overline{q} \wedge I^{p\wedge q}\overline{r}) \supset I^{p}\overline{r}$$

This principle is an instance of Cautious Transitivity:

$$(I^{\varphi}\psi \wedge I^{\varphi\wedge\psi}\eta) \supset I^{\varphi}\eta$$

A case against Cautious Transitivity

✓ In an act of imagining that Helena is on her way to meet John, she imagines that they go to the movies when they meet.

✓ In an act of imagining that Helena is on her way to meet John and they go to the movies together, she imagines that she buys some popcorn at the movie theatre.

X However, in an act of imagining that Helena is on her way to meet John, she might **not** imagine that she buys some popcorn at the movie theatre.

Summary

- Argued that AP is too strong!
- Proposed to model topic expansion via a topological closure operator. This did not change Berto's logic!
- Motivated weaker expansion operators.
- Provided definability, soundness, completeness results for weaker logics.

Further applications and future work

- Which logic is more appropriate for which theory of imagination?
- Applications of the new framework to belief, knowledge, and conditionals.
- We have not said anything about the possible worlds component of Berto's semantics. Can/Should we generalize that component as well?

Thank you!

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($S5_\Box$)	S5 axioms and rules for $\Box$
	(I) Axioms for X:
(Ax1)	$I^\psi\overline{\varphi}$ if $Var(\varphi) \subseteq Var(\psi)$
(Ax2)	$(I^\psi\varphi \wedge I^\psi\eta) \equiv I^\psi(\varphi \wedge \eta)$
(Ax3)	$I^\psi\overline{\varphi} \supset I^\eta\overline{\varphi}$ if $Var(\psi) = Var(\eta)$
(Ax4)	$I^\psi\varphi \supset I^\psi\overline{\varphi}$
	(I) Axioms connecting $\Box$ **and** I:
(Ax5)	$I^\psi\overline{\varphi} \supset \Box I^\psi\overline{\varphi}$
(Ax6)	$(I^\psi\eta \wedge \Box(\eta \supset \varphi) \wedge I^\psi\overline{\varphi}) \supset I^\psi\varphi$

Table: Axiomatization for the logic of inclusive ts-models with functions.

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Table: Axiomatization for the logic of inclusive and monotone increasing ts-models with functions.