# Multilateral Supervaluationism and Classicality

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**Goal:** Patch this hole

# (Multilateral) Supervaluationism

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  - Definitely false if false on all precisifications (*superfalse*)
  - Borderline if true on some but false on others

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- Truth = supertruth

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Graff Fara (2003) and Williamson (2018):

- Such metainferences are central to inferential practice
- SV cannot give an account of good deductive reasoning
  - Esp. without restricted versions/recapture
- SV lacks satisfying proof theory

# Multilateral Logics

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For every $\mathcal{L}$ sentence $A$, we have three *signed formulae* in $\mathcal{L}_S$

- $+A$ (strong assertion of $A$)
- $\oplus A$ (weak assertion)
- $\ominus A$ (weak rejection)

Multilateral logics are ND systems between signed formulae

# Multilateral Supervaluationism

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Within this approach, (I&S, 2022b) defined three logics:

1. SML, the basic propositional modal ($\Delta A$ = 'definitely $A$')
2. SML$^-$, slightly weaker to allow for *higher-order vagueness*
3. QSML$^-$, extension to FOL$_=$

# SML Operational Rules

$$(+\wedge\text{I.})\ \frac{+A \qquad +B}{+A \wedge B} \qquad (+\wedge\text{E.}_1)\ \frac{+A \wedge B}{+A} \qquad (+\wedge\text{E.}_2)\ \frac{+A \wedge B}{+B}$$

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$$(\ominus\neg\text{I.})\ \frac{\oplus A}{\ominus\neg A} \quad (\ominus\neg\text{E.})\ \frac{\ominus\neg A}{\oplus A} \qquad (\oplus\neg\text{I.})\ \frac{\ominus A}{\oplus\neg A} \quad (\oplus\neg\text{E.})\ \frac{\oplus\neg A}{\ominus A}$$

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$$(+\Delta\text{I.})\ \frac{+A}{+\Delta A} \qquad (+\Delta\text{E.})\ \frac{+\Delta A}{+A}$$

$$(\oplus\Delta\text{I.})\ \frac{+A}{\oplus\Delta A} \qquad (\oplus\Delta\text{E.})\ \frac{\oplus\Delta A}{+A}$$

## Coordination Principles

$$(\text{Rejection})\ \frac{+A \qquad \ominus A}{\bot} \qquad (\text{SR}_1)\ \frac{\begin{array}{c}[+A]\\ \vdots \\ \bot\end{array}}{\ominus A} \qquad (\text{SR}_2)\ \frac{\begin{array}{c}[\ominus A]\\ \vdots \\ \bot\end{array}}{+A}$$

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$$(\text{Assertion})\ \frac{+A}{\oplus A} \qquad (\text{Weak Inference})\ \frac{\oplus A \qquad \begin{array}{c}[+A]\\ +\vdots \\ +B\end{array}}{\oplus B}$$

Where $+\vdots$ means all undischarged assumptions are signed with $+$, and $(+\Delta\text{I.})$ and $(\oplus\Delta\text{I.})$ were not used

# QSML$^-$

$$(+\forall\text{I.})\ \frac{+A[a/x]}{+\forall x\, A}$$ if $a$ is any constant symbol not occurring in undischarged assumptions

$$(+\forall\text{E.})\ \frac{+\forall x\, A}{+A[a/x]}$$

$$(+=\text{I.})\ \frac{\begin{matrix}[+Fa] \\ \vdots \\ +Fb\end{matrix} \qquad \begin{matrix}[+Fb] \\ \vdots \\ +Fa\end{matrix}}{+a = b}$$ where $F$ is a predicate symbol not occurring in undischarged assumptions

$$(+=\text{E.}_1)\ \frac{+a = b \qquad +Fa}{+Fb} \qquad (+=\text{E.}_2)\ \frac{+a = b \qquad +Fb}{+Fa}$$

# Restricted Rules in MSV

**The MSV systems derive:**

$$(+\to\text{I.})\ \frac{\begin{array}{c}[+A]\\ +\vdots\\ +B\end{array}}{+A\to B}\qquad (+\vee\text{E.})\ \frac{+A\vee B\qquad \begin{array}{c}[+A]\\ +\vdots\\ +C\end{array}\qquad \begin{array}{c}[+B]\\ +\vdots\\ +C\end{array}}{+C}$$

$$(+\neg\text{E.})\ \frac{\begin{array}{c}[+\neg A]\\ +\vdots\\ \bot\end{array}}{+A}\qquad (+\exists\text{E.})\ \frac{+\exists x\, A\qquad \begin{array}{c}[+A[a/x]]\\ +\vdots\\ +B\end{array}}{+B}$$

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if $a$ is any constant symbol not occurring in $A$, $B$ or undischarged assumptions

**(I&S, 2022b)**: this solves Graff Fara and Williamson's challenge

- We have simple, harmonious proof theory
- With restricted versions of the invalid classical metarules

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- We cannot establish on which levels SML, $\text{SML}^-$, $\text{QSML}^-$ behave classically
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- We cannot establish on which levels SML, SML$^-$, QSML$^-$ behave classically
  - We don't know which departures we have to justify/explain
  - Or whether all differences are SV-*necessary*
- It is unclear how (I&S)'s derived rules actually relate to the classical principles they are supposed to refine

# Goals

1. Develop a method for comparing valid principles of any given level (theorems, inferences, metainferences, metametainferences, ...) between uni- and multilateral logics

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3. Investigate potential for SV-acceptable classicality improvements

4. Reassess the response to Graff Fara and Williamson

# Measuring Classicality

# Inferential Levels and Validity

## Definition

Let $\mathcal{L}$ be some formal language, with $\mathcal{L}^0$ its set of wff.

$$\mathcal{L}^{n+1} := \{\langle \Gamma, \Psi \rangle \mid \Gamma \cup \{\Psi\} \subseteq \mathcal{L}^n\}.$$

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**Level 1**

$$p \vee q, \neg q \Rightarrow^1 r \rightarrow p$$

**Level 2**

$$\frac{p \vee q, \neg q \Rightarrow^1 r \rightarrow p}{p \Rightarrow^1 q \rightarrow p}\ 2$$

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## Definition

- $\Gamma \Rightarrow^1 \Psi$ is **valid** iff $\Gamma \vdash \Psi$
- $\Gamma \Rightarrow^{n+1} \Psi$ is **valid** iff either some $\gamma \in \Gamma$ is not valid, or $\Psi$ is valid (Global validity)

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E.g. in the sentential (not quantified) setting:

**Definition**

Take a set of metalinguistic variables $\mathcal{A} = \{A_1, A_2, A_3, ...\}$, and let $\mathcal{A}^B$ be it's closure of under $\neg$ and $\wedge$. The set $UBS^n$ of level n schemas is:

$$UBS^1 := \{\langle \Lambda, \Omega \rangle \mid \Lambda \cup \{\Omega\} \subseteq \mathcal{A}^B\}.$$

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**Valid** in logic $\mathcal{K}$ on language $\mathcal{L}$ iff valid for all substitutions $\sigma : \mathcal{A} \to \mathcal{L}^0$

**But:** this assumes $\mathcal{L}^0$ is closed under $\neg$ and $\wedge$

# The Substitution Issue

(Conjunction Elimination): $A_1, A_2 \wedge A_3 \Rightarrow^1 A_2$

**Q:** What are its instances in a multilateral language?

1. Let $A_1, A_2, A_3$ range over signed formulae
   - $+r, (+p) \wedge (+q) \Rightarrow^1 +p$ ?
2. Let them range over sentences
   - $r, p \wedge q \Rightarrow^1 p$ ?
3. Let $A_1, A_2, A_3$ range over sentences, *then* add force-markers. **But how?**

3a. Include all combinations of force markers
   - $+r, +(p \wedge q) \Rightarrow^1 \ominus p$ ?

3b. Include only 'uniform' ones (prefixing the same sign to every sentence)
   - $\ominus r, \ominus(p \wedge q) \Rightarrow^1 \ominus p$ ?

3c. Only apply $+$
   - $\ominus r, +(p \wedge q) \Rightarrow^1 +p$

# Multilateral Schemas

**Solution:** Define a separate notion of *multilateral schema*

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Let $\mathcal{A}^B$ be $\mathcal{A}$'s closure under $\neg$ and $\wedge$, and $\mathcal{A}^+ = \{+X | X \in \mathcal{A}^B\}$.

$$MBS^1 := \{\langle \Lambda, \Omega \rangle \mid \Lambda \cup \{\Omega\} \subseteq \Phi \cup \mathcal{A}^+\}$$

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**Valid** in $\mathcal{K}$ on language $\mathcal{L}_S$ iff valid for all substitutions $\sigma = \sigma_\mathcal{A} \cup \sigma_\Phi$, with $\sigma_\mathcal{A} : \mathcal{A} \to \mathcal{L}^0$ and $\sigma_\Phi : \Phi \to \mathcal{L}^0_S$

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- We can express rules that combine these aspects, such as (Multilateral Reductio):

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Which has instances such as:

$$\frac{\oplus p, +\neg p \Rightarrow \bot}{\oplus p \Rightarrow +p}$$

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**(MR):** **(CR):**

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**(CR):**

**So:** MSV systems depart from classical logic at level 2, because they invalidate (MR), while classical logic validates U[(MR)] = (CR)

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## Multilateral Classicality

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**A**: When any level $n$ multilateral schema is valid in $\mathcal{K}$ iff its unilateralization is classically valid.

**Moreover:** $\mathcal{K}$ is strictly weaker/stronger than classical logic on $n$ when the entailment only goes in one direction

# Results

# SML and SML$^-$

**Theorem**

Both SML and SML$^-$ are:

- Precisely classical on level 1
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Any logic at least as strong as SML$^-$ but classical at level 2 won't allow for borderline cases:

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**Conclusion:** Failure of metainferences is *necessary* for the (multilateral) supervaluationist

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**Theorem**

Any ND which derives the rules of QSML$^-$ but is classical on level 1 doesn't leave room for higher-order vagueness:

- Related to work by Graff Fara (2003) and Zardini (2013)

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**Theorem**

Any ND which derives the rules of $\text{QSML}^-$ but is classical on level 1 doesn't leave room for higher-order vagueness:

- Related to work by Graff Fara (2003) and Zardini (2013)

**Conclusion:** In $\text{FOL}_=$, failure of classical theorems is *necessary* for the (multilateral) supervaluationist

# Conclusion

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2. Failure of (MR) was *necessary*
3. We can relate (I&S, 2022b) derived restricted rules to the respective classical principles
   Recall the question how e.g. (+¬E.) relates to *reductio*

$$
\begin{array}{c}
[+\neg A] \\
+\vdots \\
(+\neg\text{E.})\ \dfrac{\bot}{+A}
\end{array}
$$

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   Recall the question how e.g. $(+\neg\text{E.})$ relates to *reductio*

$$(+\neg\text{E.})\ \dfrac{\begin{matrix}[+\neg A]\\ +\vdots\\ \bot\end{matrix}}{+A} \quad \xrightarrow{\textit{Restrict.}} \quad (+\neg\text{E.}^{*})\ \dfrac{\begin{matrix}[+\neg A]\\ \vdots\\ \bot\end{matrix}}{+A}$$

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3. We can relate (I&S, 2022b) derived restricted rules to the respective classical principles
   Recall the question how e.g. $(+\neg\text{E.})$ relates to *reductio*

$$(+\neg\text{E.})\ \frac{\begin{array}{c}[+\neg A]\\ +\vdots\\ \bot\end{array}}{+A} \xrightarrow{\textit{Restrict.}} (+\neg\text{E.}^*)\ \frac{\begin{array}{c}[+\neg A]\\ \vdots\\ \bot\end{array}}{+A} \xrightarrow{\textit{Admis./Val.}} \frac{\varphi, +\neg A \Rightarrow \bot}{\varphi \Rightarrow +A}$$

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$$\frac{A_1, \neg A_2 \Rightarrow \bot}{A_1 \Rightarrow A_2} \xrightarrow{\mathit{Expresses}} \textbf{Reductio}$$

# First-order Acceptability

For QSML$^{-}$:

1. We now know that the failure of the metainferences is **not** the only departure from classicality we need to answer for
2. The failure of $=$-substitution raises a new instance of Acceptability
   - (I &S, 2022b) suggest a contextual reading: different ways of referring to one object may be associated with different standards of definite tallness/darkness/...
   - This reading justifies failure of $=$-substitution in $\Delta$-contexts
3. Results that $=$-substitution is the only level 1 departure from classicality to answer for
4. It is necessary to account for higher-order vagueness

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- Understanding/intuitions about unilateral logic can be brought to bear on the multilateral setting
- SV and classical logic are a case study

# References

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