Entailment and Containment: a Ternary Approach to Information and Topic Inclusion

Thomas M. Ferguson [1] Pietro Vigiani [2]

[1]Rensselaer Polytechnic Institute, Department of Cognitive Sciences

[2]Scuola Normale Superiore, Department of Philosophy

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- Consider Relevant Containment Logics.

Plan of work

0. Relevant logic
   Information inclusion via contextual entailment.
1. Containment logics on the market
   What do they miss.
2. Ternary Relevant Containment Logic
   Contextual information and topic inclusion.

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- $V : At \to \textit{Prop}$ — Valuation
- $L \subseteq \{s \mid X \sqsubseteq_s Y\}$ iff $X \subseteq Y$ — Semantic deduction theorem

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Interpretation and Validity

- Language $\langle \mathbf{t}, \neg, \wedge, \vee, \rightarrow \rangle$ interpreted as expected:

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[\![\mathbf{t}]\!]_{\mathfrak{M}} &= L \\
[\![\neg\varphi]\!]_{\mathfrak{M}} &= \{s \mid s^* \notin [\![\varphi]\!]_{\mathfrak{M}}\} \\
[\![\varphi \wedge \psi]\!]_{\mathfrak{M}} &= [\![\varphi]\!]_{\mathfrak{M}} \cap [\![\psi]\!]_{\mathfrak{M}} \\
[\![\varphi \vee \psi]\!]_{\mathfrak{M}} &= [\![\varphi]\!]_{\mathfrak{M}} \cup [\![\psi]\!]_{\mathfrak{M}} \\
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- Validity is truth at all normal states $L \subseteq S$

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  1. Truth component — S4 Kripke model
  2. Topic component — Topic model

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- Fine's result: $\vdash_{\mathsf{PAI}} \varphi \twoheadrightarrow \psi$ *iff* $\begin{cases} \vdash_{\mathsf{S4}} \Box(\varphi \supset \psi) \\ At(\psi) \subseteq At(\varphi). \end{cases}$

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- Why variable inclusion? Syntactic filter on S4 strict implication is not meaning containment.

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- In relevant logic there is a discontinuity between normal and non-normal states in the analysis of entailment:
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- Similarly for meaning containment:
  1. containment in normal states is assessed with Fine's topic models.
  2. containment in non-normal states is assessed more generally.

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- Validity as in relevant logic (truth in all normal states).

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- This discontinuity is not only artifical but it gives also no conceptual insight on the meaning of containment.

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$a \preceq_s b$:= *the topic* a *is contained in topic* b *according to the discursive context fixed by* s.

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- Relativising to a discursive context can restrict or widen the number of admissible content inclusions.

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- $(a \rightarrow_s b) \wedge_s (a \supseteq_s b) = a \twoheadrightarrow_s b$ $\varphi \twoheadrightarrow \psi := \varphi \rightarrow \psi \wedge \varphi \supseteq \psi$

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$$a \vee_l b = b \vee_l a$$
$$(a \vee_l b) \vee_l c = a \vee_l (b \vee_l c)$$
$$a \vee_l a = a$$
$$a \vee_l b = a \wedge_l b$$
$$a \preceq_l b \text{ iff } a \vee_l b = b$$
$$\neg_l a = a$$

Ternary containment models - 3

- Topic content $(\!|\varphi|\!)$ defined recursively: for $\circledast \in \{\neg, \wedge, \vee, \rightarrow, \supseteq, \twoheadrightarrow\}$

$$(\!|p|\!)^s_{\mathfrak{M}} = \tau_s(p)$$
$$(\!|\circledast(\varphi_1, \ldots, \varphi_n)|\!)^s_{\mathfrak{M}} = \circledast_s((\!|\varphi_1|\!)^s_{\mathfrak{M}}, \ldots, (\!|\psi|\!)^s_{\mathfrak{M}})$$

Ternary containment models - 3

- Topic content $(\!|\varphi|\!)$ defined recursively: for $\circledast \in \{\neg, \wedge, \vee, \rightarrow, \supseteq, \twoheadrightarrow\}$

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- $(\!|\varphi|\!)$ determines information content $[\![\varphi]\!]$:

$$[\![p]\!]_{\mathfrak{M}} = V(p)$$
$$[\![\circledast(\varphi_1, \ldots, \varphi_n)]\!]_{\mathfrak{M}} = \circledast([\![\varphi_1]\!]_{\mathfrak{M}}, \ldots, [\![\psi]\!]_{\mathfrak{M}})$$
$$[\![\varphi \supseteq \psi]\!]_{\mathfrak{M}} = \{s \mid (\!|\psi|\!)^s_{\mathfrak{M}} \preceq_s (\!|\varphi|\!)^s_{\mathfrak{M}}\}$$

Properties

- By our semantics:

$$s \models \varphi \twoheadrightarrow \psi \Leftrightarrow \begin{cases} [\![\varphi]\!] \sqsubseteq_s [\![\psi]\!] \\ (\!|\psi|\!)^s \preceq_s (\!|\varphi|\!)^s. \end{cases}$$

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Lemma 1 (Entailment and Containment)

$\mathfrak{M} \models \varphi \rightarrow \psi$ *iff* $[\![\varphi]\!]_{\mathfrak{M}} \subseteq [\![\psi]\!]_{\mathfrak{M}}$
$\mathfrak{M} \models \varphi \supseteq \psi$ *iff* $\forall l \in L((\!|\psi \vee \varphi|\!)^l_{\mathfrak{M}} = (\!|\varphi|\!)^l_{\mathfrak{M}})$.

Axiomatisation of TRC

- Ternary relevant containment logic can be axiomatised as follows:
  1. Axiom and rules for relevant propositional logic FD(N)E.
  2. Axioms for containment formulas:

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\begin{array}{ll}
\text{(C1)} & \varphi \supseteq \varphi \\
\text{(C2)} & \varphi \equiv \neg\varphi \\
\text{(C3)} & (\varphi \wedge \psi) \supseteq \varphi(\psi) \\
\text{(C4)} & (\varphi \vee \psi) \equiv (\varphi \wedge \psi) \\
\text{(C5)} & \mathbf{t} \wedge (\varphi \supseteq \psi) \wedge (\psi \supseteq \chi) \rightarrow (\varphi \supseteq \chi) \\
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- N.B. Contrary to (Fine1986) and (Sylvan1988), it does not contain

$$\varphi \supseteq \psi \quad \textit{if } At(\psi) \subseteq At(\varphi).$$

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- For every CFD(N)E-theory s, we take $\sim_s$ as the reflexive closure of:

$$\varphi \sim'_s \psi \text{ iff } \mathbf{t} \wedge (\varphi \supseteq \psi) \wedge (\psi \supseteq \varphi) \in s$$

Generalised canonical topic model

$$\mathfrak{T}^c_s = (\mathcal{T}_s, \preceq_s, \neg_s, \vee_s, \wedge_s, \rightarrow_s, \supseteq_s, \twoheadrightarrow_s, \tau_s)$$

- $\mathcal{T}_s = \mathcal{L}/\sim_s$;
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Theorem 1

$\models \varphi \Leftrightarrow \vdash_{\mathsf{TRC}} \varphi$

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  2. ψ is topically included in φ
- Both information and topic inclusion are ternary relations, relativised to some information state.
- Aligns well with intuition that entailment and containment considerations are evaluated in situ (wrt a situation).